070301 Quiz 3 Nanopowders

We have discussed transport and thermodynamics of nanoparticles in low density media for the past few weeks.

1) For a droplet of liquid, such as a molten iron oxide particle formed in a flame, the surface tension γ serves to compress the liquid compared to the gas in which the droplet is immersed. Calculate the pressure difference by considering the addition of a small amount of material to the droplet, dv, and its effect on the added surface differential element and the added volumetric differential element. If a small droplet touches a large droplet which droplet would grow?

2) The Kelvin equation restates the Gibbs-Thompson equation for a droplet in equilibrium with a vapor. The Gibbs-Thompson equation given previously was,

$$\ln(S) = \frac{2\gamma}{rkT}$$

where S is ϕ/ϕ_0 and ϕ is the mole fraction of a nucleating component. Use the ideal gas law and the definition of ϕ for a gas as n/V to obtain the Kelvin Equation. Sketch $\ln(p/p_s)$ for the Kelvin equation versus particle size r and show how this would deviate in the presence of a salt.

For a magnetic nanoparticle, such as the iron oxides magnetite or maghemite, the motion of a particle in a magnetic field is governed partly by the magnetic force,

$$F_{mag} = \frac{4\pi}{3}\left(\frac{d_p}{2}\right)^3 M_{sat}\frac{dB}{dx}$$

where dB/dx is the magnetic field gradient, M_{sat} is the saturation magnitization of the nanoparticle and d_p is the Sauter mean diameter.

3) How would you calculate the terminal velocity of a particle subject to F_{mag} using the friction factor, f? Give the scaling relationship between terminal velocity and size, d_p, for the free molecular regime and for the continuum regime.

4) In considering nanoparticles subject to Brownian motion we calculate the flux J_x which is the mass velocity (velocity times mass) using Fick's first law. Give Ficks first law then add the mass velocity due to the applied magnetic field to obtain a differential expression for the mass flux under a magnetic field. Substitute the Stokes-Einstein expression for the diffusion coefficient and the two definitions of the friction factor for free molecular and continuum conditions to obtain two expressions for the flux.

5) The solution to the differential expression for steady state (J_x constant) is,

$$J_x = \frac{-c_{mag} n_b}{1 - \exp\left(-\frac{c_{mag} b}{D}\right)}$$

where b is the diffusion layer thickness. Show that this expression reduces to a magnetically driven and a diffusive limit. What is the particle size dependence in these two regimes?

1)

$(4\pi r^2 dr) = dv$

$(8\pi r dr) = dS$

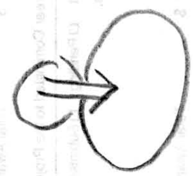
$(P_L - P_V) dv = \gamma dS$

$(P_L - P_V)(4\pi r^2 dr) = \gamma (8\pi r dr)$

$\Delta p r = 2\gamma$

$\Delta p = \frac{2\gamma}{r}$

a small droplet has a much higher pressure



Large droplet grows.

(2)

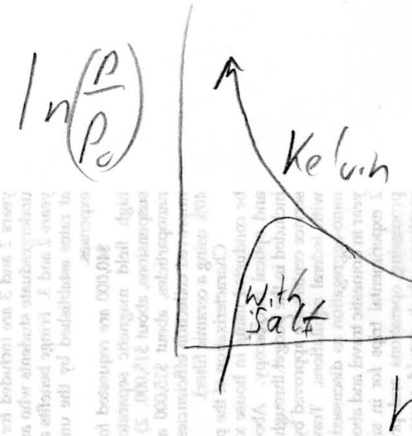
Ideal Gas law

$$p = \left(\frac{n}{V}\right) kT$$

$$\frac{n}{V} = \phi \text{ for a gas } = \frac{p}{kT}$$

$$\frac{\phi}{\phi_0} \Rightarrow \left(\frac{p}{p_0}\right)$$

$$\ln\left(\frac{p}{p_0}\right) = \frac{2\gamma}{rkT} \quad \text{Kelvin Eqn}$$



Salt

$$\ln \frac{p}{p_{s0}} = \frac{2\gamma}{rkT} - \frac{3 n_2}{4\pi r^3}$$

n_2 = # of solvated molecules

(3)

$$F_{mag} = f\, C_{mag}$$

$f = 3\pi d_p \eta_0$ Stokes–Continuum

or

$f \sim d_p^2$ Free Molecular

$$C_{mag} = \frac{F_{mag}}{f} \qquad C_{mag} \sim \frac{d_p^3}{d_p} \sim d_p^2 \text{ Continuum}$$

$$C_{mag} \sim \frac{d_p^3}{d_p^2} \sim d_p \text{ Free Molecular}$$

(4)

$$J_x = -D\frac{dn}{dx} \quad \text{Fick's First Law}$$

$$J_x = -D\frac{dn}{dx} + C_{mag}\, n$$

Stokes Einstein $\boxed{D = \frac{kT}{f}} \rightarrow D = \frac{kT}{3\pi d_p \eta_0}$ Continuum

$$D = \frac{kT}{C d_p^2} \quad \text{Free Molecular}$$

Continuum

$$J_x = \frac{-kT}{C_1 d_p^2}\frac{dn}{dx} + C_2 d_p n \quad \text{Free Molecular}$$

$$J_x = \frac{-kT}{3\pi d_p \eta_0}\left(\frac{dn}{dx}\right) + C_3 d_p^2 n \quad \text{Continuum}$$

(5)

$$J_x = \frac{-C_{mag}\, n_b}{1 - \exp\left(-\frac{C_{mag} b}{D}\right)}$$

if $C_{mag} b \ll D$

$$\exp(-x) \xRightarrow{x \text{ small}} (1-x)$$

$$J_x = \frac{-C_{mag}\, n_b}{(1-1) + \left(\frac{C_{mag} b}{D}\right)} = -\frac{n_b D}{b}$$

→ Continuum $J_x \sim \frac{1}{d_p}$

→ $J_x \sim \frac{1}{d_p^2}$ Free Molecular

Diffusion Governs

if $D \ll C_{mag} b$

$$\exp\left(-\frac{C_{mag} b}{D}\right) \Rightarrow 0$$

$$J_x = -C_{mag}\, n_b$$

→ Continuum $J_x \sim d_p^2$

→ $J_x \sim d_p$ Free Molecular

Magnetic Field Dominated